Inclusion of theory errors in PDF determination with ML techniques

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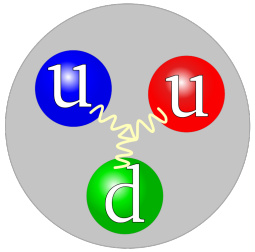
Outline

1. **Introduction and motivation**
   - What are parton distribution functions (PDFs)?
   - Why do we need them?
   - What do we mean with theory errors?
   - How can we estimate them?
2. **PDF determination**
   - Why do we use machine learning?
   - The NNPDF methodology.
3. **Inclusion of theory errors**
   - How can we include theory errors?
   - Impact of theory errors in the fit.

Introduction and motivation

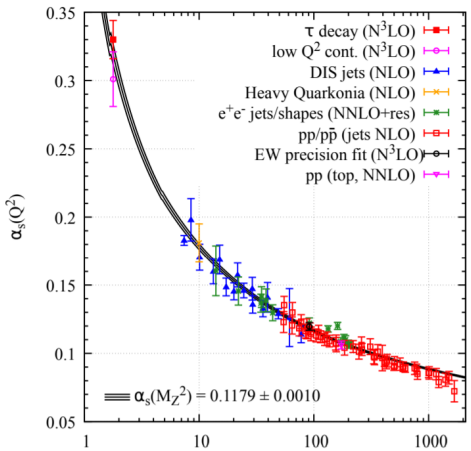
Quantum Chromodynamics (QCD)

- QCD: theory of the strong interactions.
- Fundamental fields: quarks and gluons (*partons*).
- Binds quarks to form hadrons.
  - Perturbative expansions in terms of the *strong coupling constant* α_s.



$$\hat{\sigma} = C^{(0)} + \alpha_s C^{(1)} + \alpha_s^2 C^{(2)} + \mathcal{O}(\alpha_s^3)$$

- *Asymptotic freedom:*



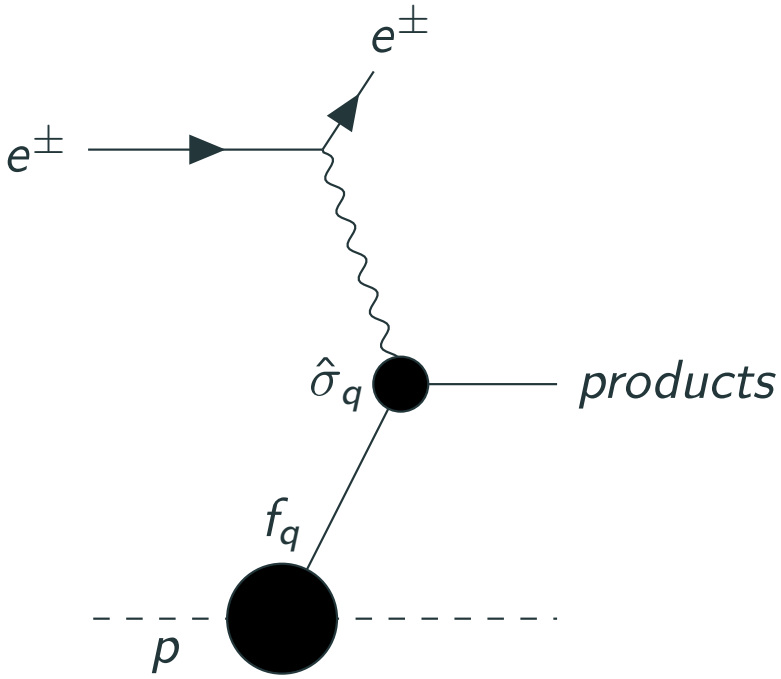
Parton distribution functions (PDFs)

- Partonic (high energy) interaction described by $\hat{\sigma}_q$.
- Internal dynamics of the proton (low energy) described by the PDF f_q.

Factorization theorem:

$$\sigma_X = \sum_q \overbrace{f_{q/p}}^{\text{PDFs}} \otimes \overbrace{\hat{\sigma}_{q\gamma\to X}}^{\text{theory}}$$

Deep Inelastic Scattering



PDF fit:

- σ_X^D: measured by experiments.
- σ_X^P: predicted using $\hat{\sigma}_{q\gamma\to X}$
- minimization of $\chi^2 = f(\sigma_X^D - \sigma_X^P, C)$

Theory errors: MHOU

$$\hat{\sigma} = C^{(0)} + \alpha_s C^{(1)} + \alpha_s^2 C^{(2)} + \overbrace{\mathcal{O}(\alpha_s^3)}^{\text{MHOU}}$$

- Theory errors: *missing higher orders uncertainty*.
- Cannot be neglected in a PDF fit:
  - MHOU $\approx$ uncertainties of current PDF determinations.
  - MHOU affects relative weights of observables.
- Estimated from the dependence of observables on certain parameters.

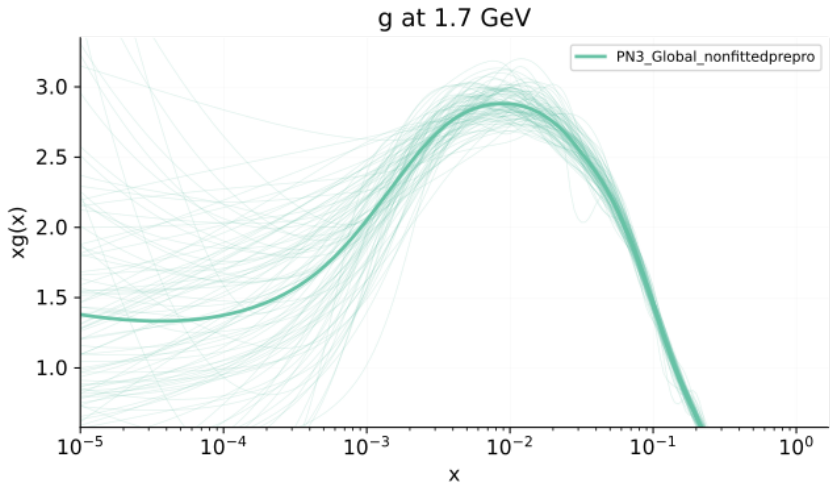
$$\sigma^P(\mu') - \sigma^P(\mu) = \mathcal{O}(\alpha_s^3)$$

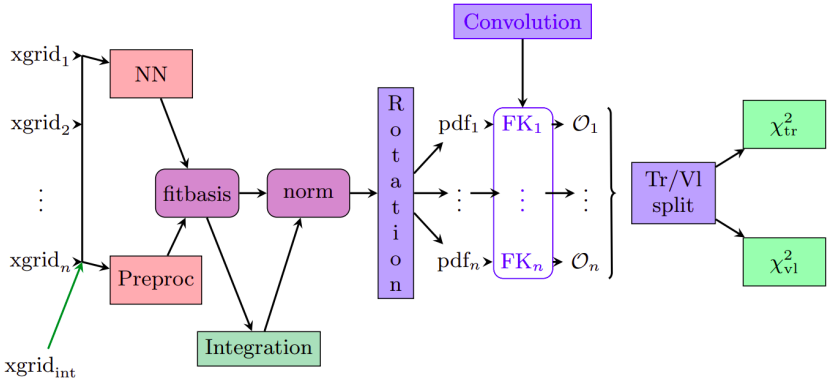
PDF determination

The fitting problem: NNPDF methodology

- Neural Network used to provide an unbiased functional form
  - $f_i = \mathrm{A}_i x^{\alpha_i}(1-x)^{\beta_i}\mathrm{NN}_i(x, \log x)$
- Minimization of the *loss function*

$$\chi^2 = \sum_{ij}^{N_{\mathrm{dat}}} (D-P)_i C_{ij}^{-1} (D-P)_j \quad C = \text{experimental covariance matrix}$$





Inclusion of theory errors

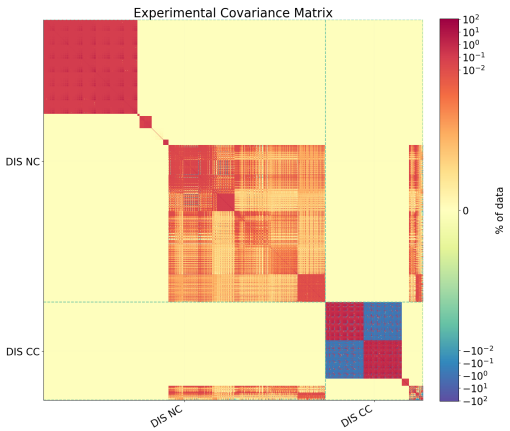
How to include them

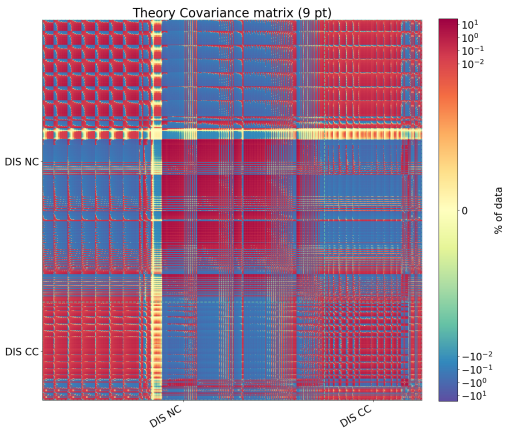
- *Theory covariance matrix*.

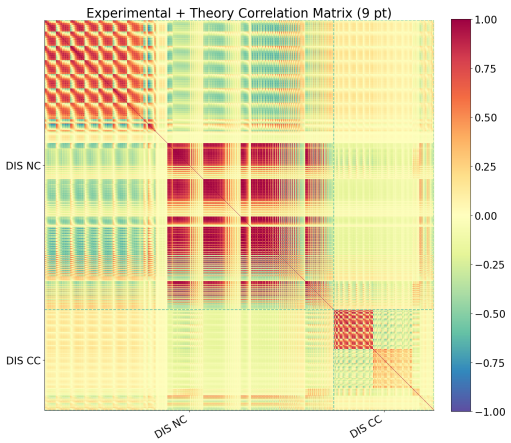
$$S_{ij} = n_m \sum_{V_m} (P_i(\mu_m) - P_i(0))(P_j(\mu_m) - P_j(0))$$

- Including it in the *loss function*.

$$\chi^2 = \sum_{ij}^{N_{\text{dat}}} (D - P)_i (C + S)_{ij}^{-1} (D - P)_j$$

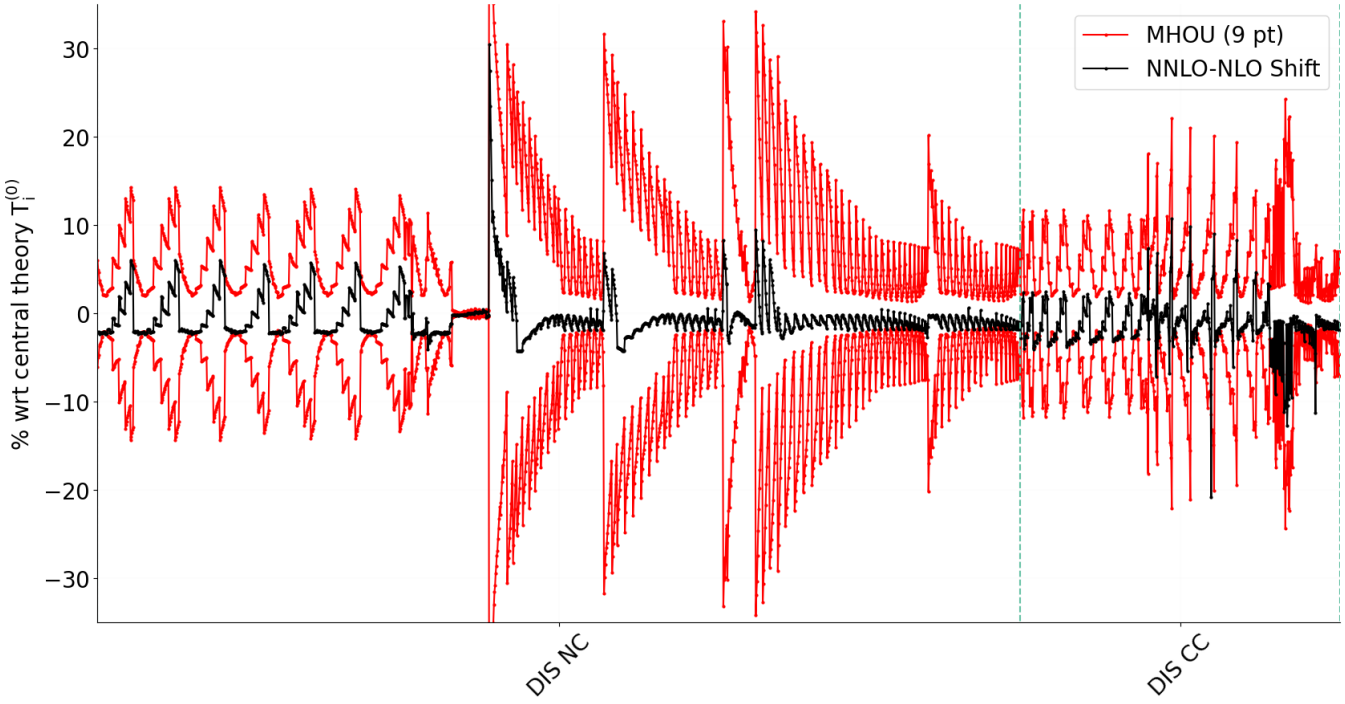






Validation of the estimation

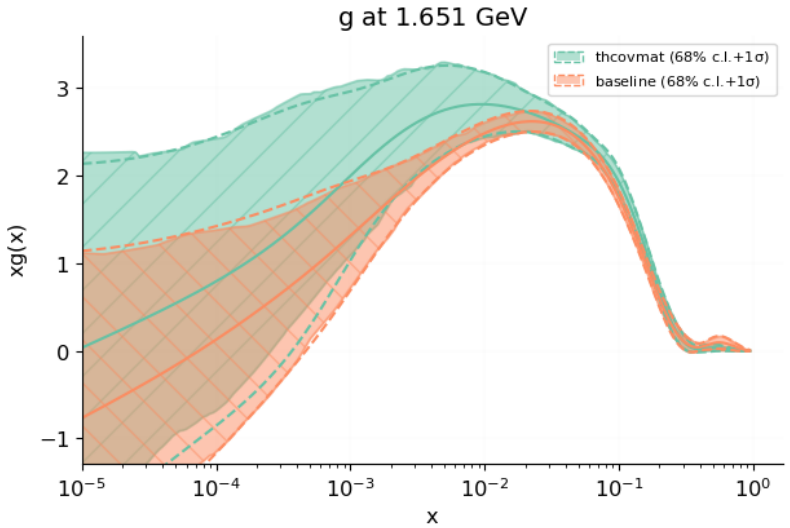
- Most of the predictions are currently known up to NNLO ($\mathcal{O}(\alpha_s^2)$).
- Compare NNLO – NLO shifts with MHOU on NLO ($\mathcal{O}(\alpha_s)$).

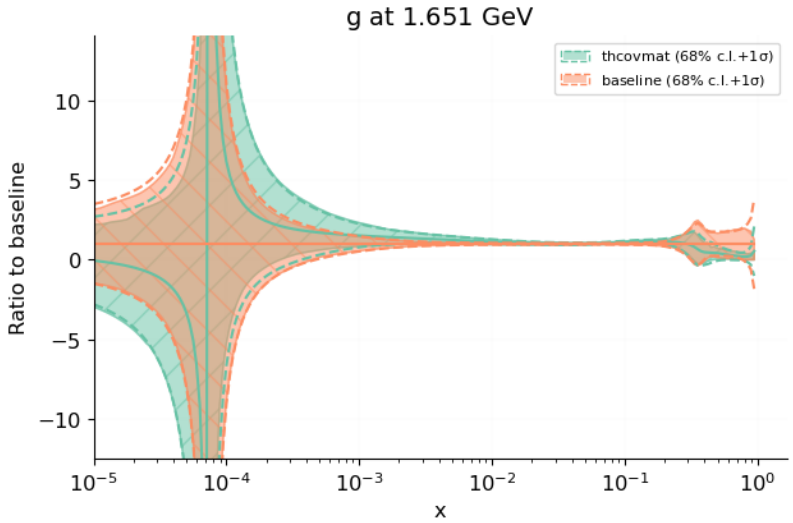


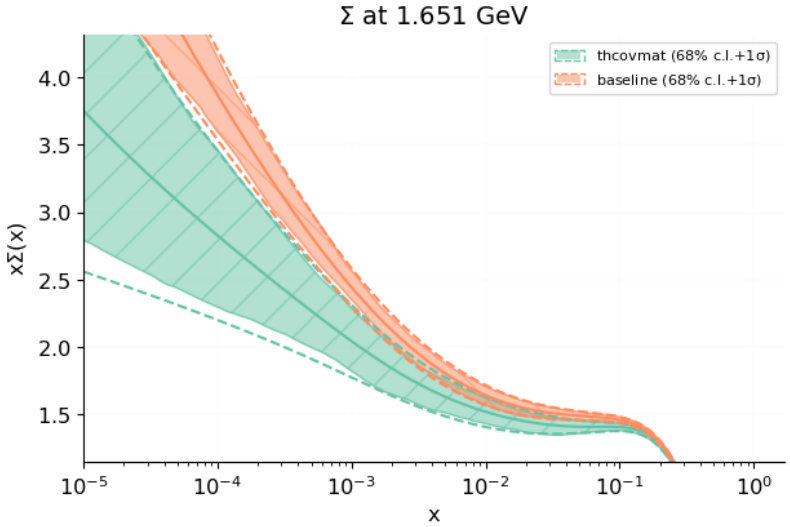
Impact on the fit: NLO results

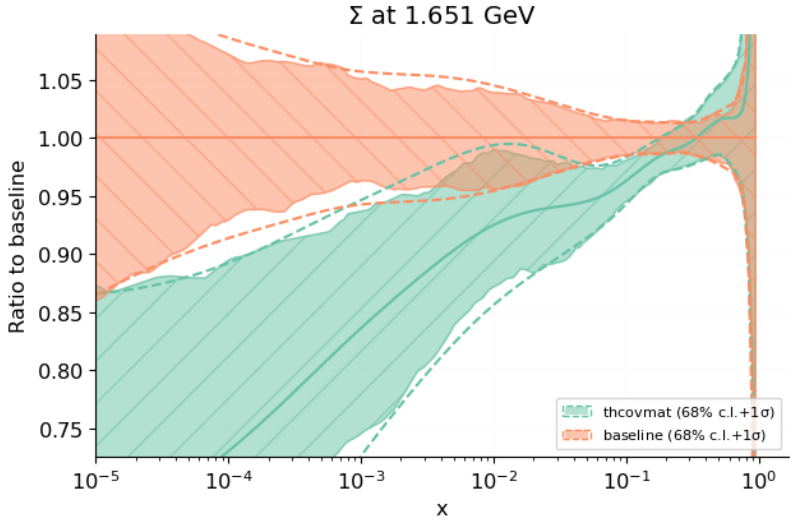
Absolute

Ratio









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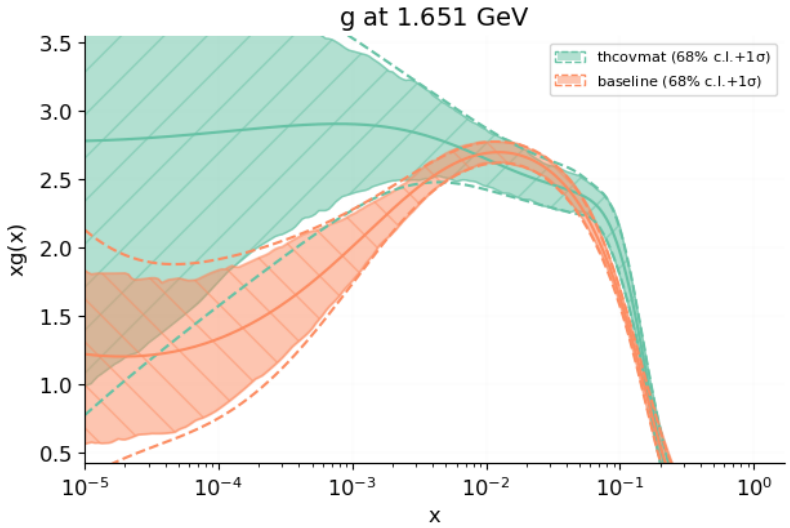

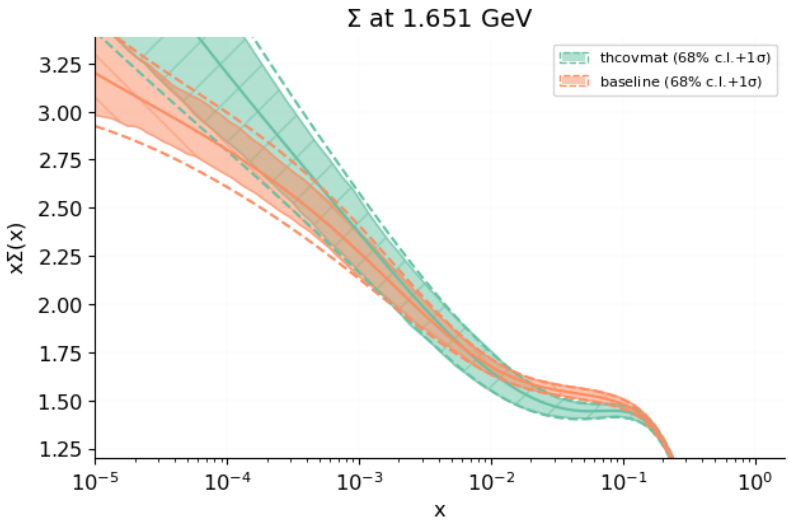

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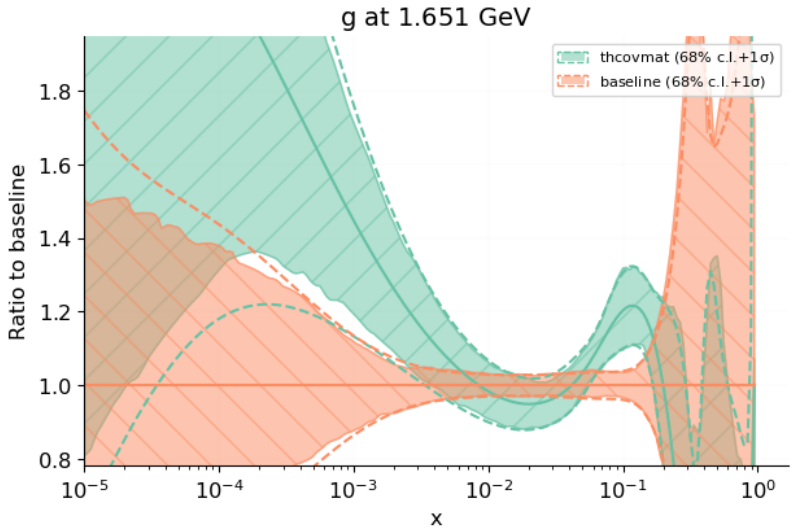

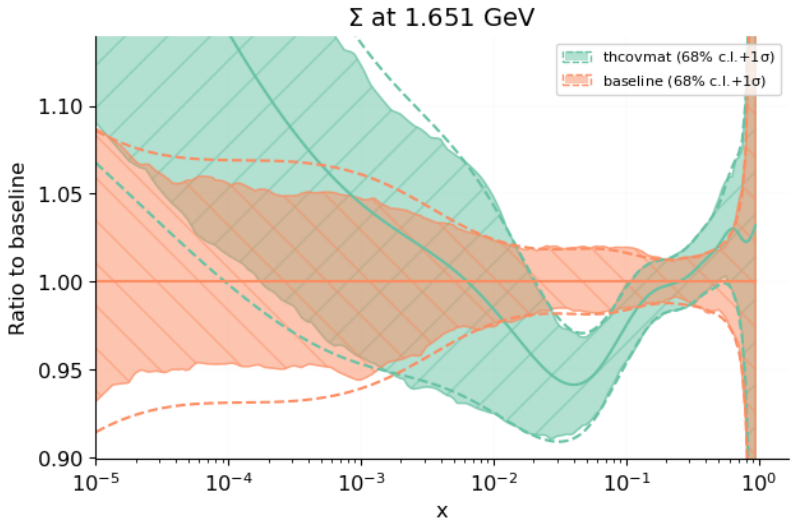

Conclusions

- An unbiased and trustworthy determination of PDFs and of their uncertainties an be achieved trough advanced ML techniques.
- The determination can be made even more trustworthy if theory errors are included.
- The results obtained including theory errors show that they change both the uncertainties and the central value of the PDFs in a non negligible way.

Thanks for your attention!