



The inclusion of theory errors in PDF fitting.

The NNPDF4.0MHOU PDFs set

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Outline.



INTRODUCTION

- What are theory errors?
- How can we estimate them?
- Why is it relevant to include them in a PDF fit?



METHODOLOGY AND VALIDATION

- How can we include MHOU in a PDF fit?
- Can we validate our estimation?



RESULTS

- What is the impact on the PDFs at NLO and NNLO?
- What is the impact on the phenomenology?

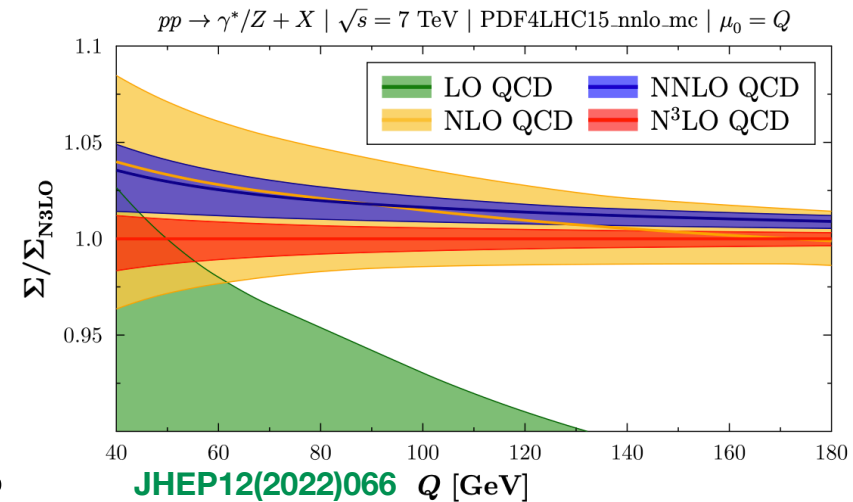
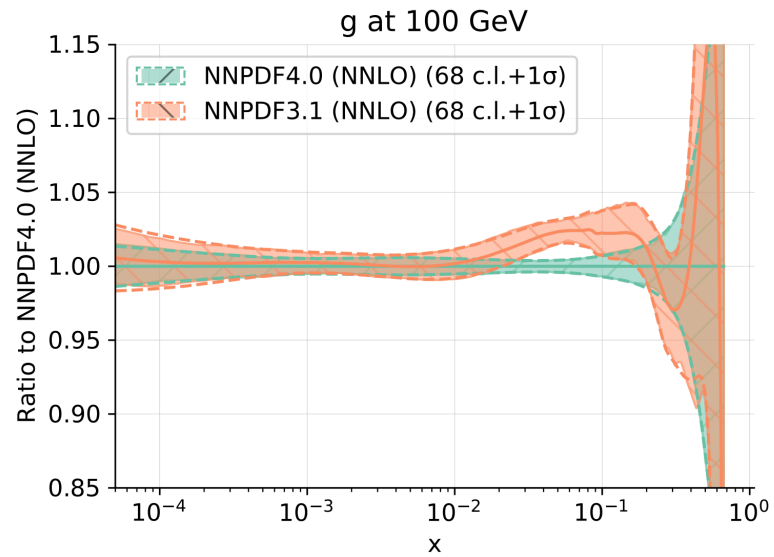
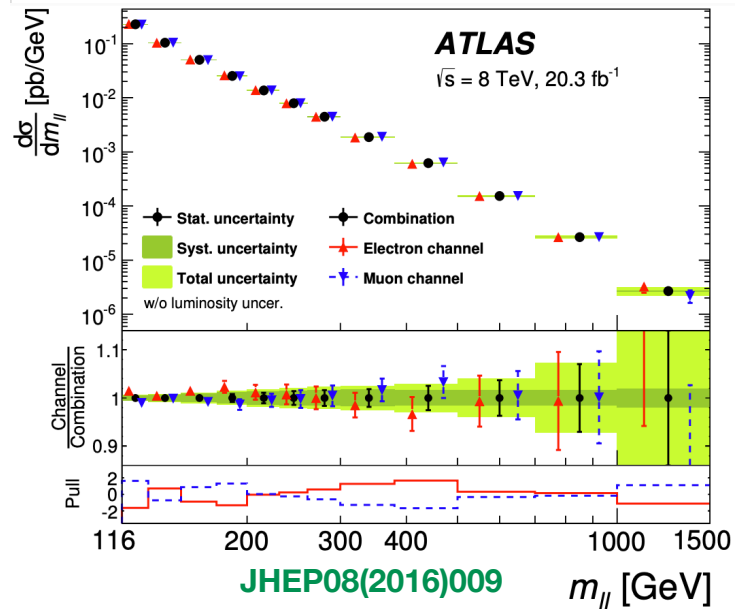
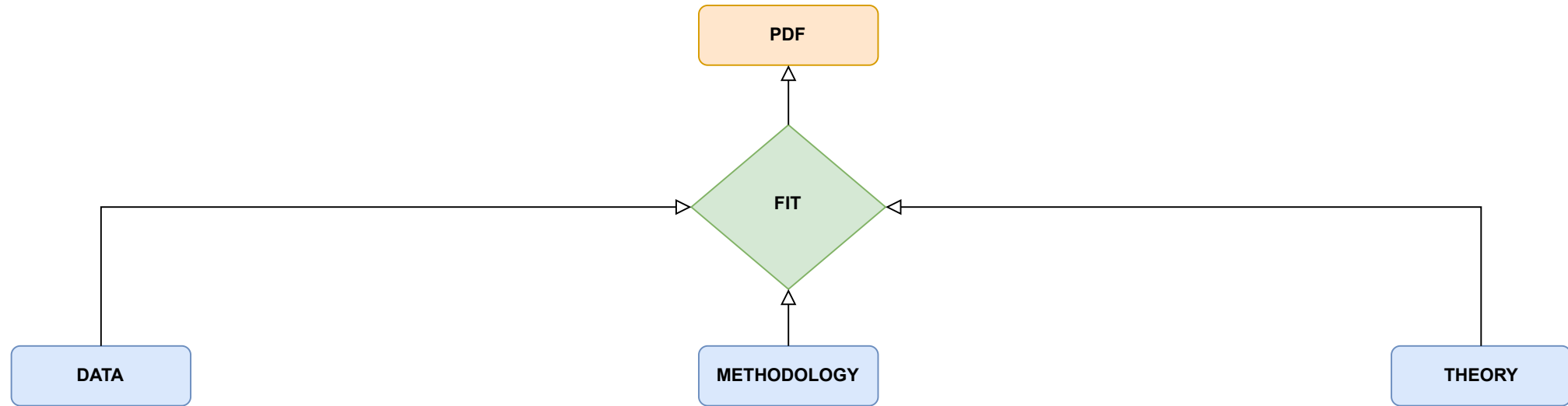


INTRODUCTION

- What are theory errors?
- How can we estimate them?
- Why is it relevant to include them in a PDF fit?

*“We are not strangers, only the introduction is missing”
(Jesus Apolinaris)*

Motivation.

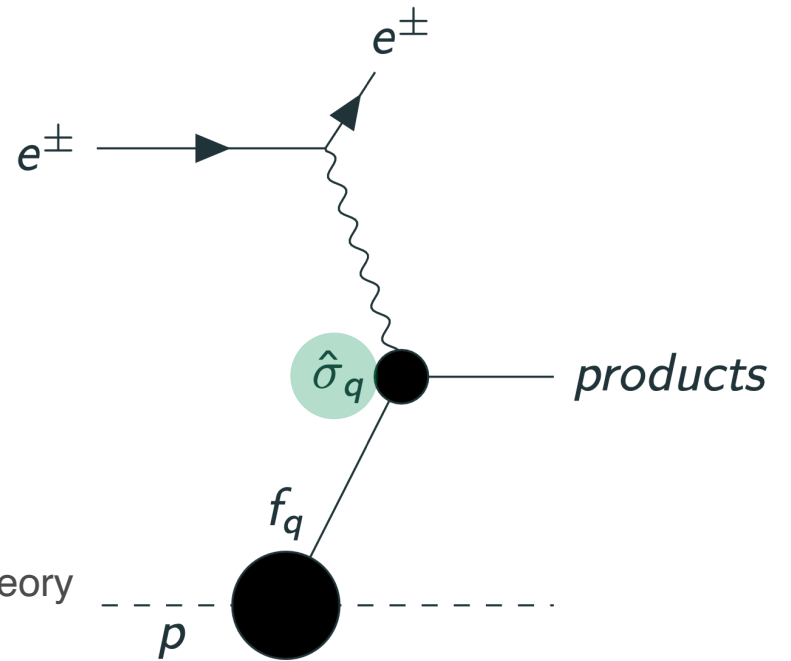


Definition of the problem.

$$F(Q) = \hat{\sigma}(Q) \otimes U(Q, Q_0) \otimes f(Q_0)$$

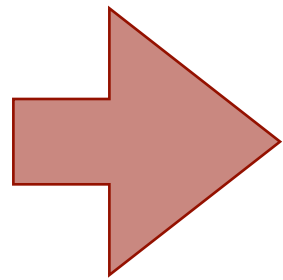
- **Partonic cross sections** are computed in perturbation theory
- **DGLAP evolution operator** evolves the PDFs from Q_0 to Q
- **Anomalous dimensions** inside DGLAP operator are computed in perturbation theory

Deep Inelastic Scattering (DIS)



$$\hat{\sigma}^{NLO} = \hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \mathcal{O}(\alpha_s^2)$$

$$\gamma^{NLO} = \alpha_s \gamma^{(0)} + \alpha_s^2 \gamma^{(1)} + \mathcal{O}(\alpha_s^3)$$



MHOU
(Missing Higher Order Uncertainties)

How can we estimate them?

Theory errors: estimation.

Scale Variations

$$\bar{F}^{NLO}(\mu_f = \kappa_f Q, \mu_r = \kappa_r Q) - F^{NLO}(\mu_f = Q, \mu_r = Q) = \mathcal{O}(NNLO)$$

Factorization scale

Estimates **MHOU** of anomalous dimensions

$$U^{NLO}(Q, Q_0) \rightarrow \bar{U}^{NLO}(Q, Q_0, \kappa_f)$$

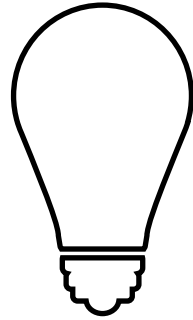
Renormalization scale

Estimates **MHOU** of partonic cross sections

$$\hat{\sigma}^{NLO}(Q) \rightarrow \bar{\sigma}(Q, \kappa_r)$$



$\kappa_f, \kappa_r \in (0.5, 2.0)$ is the most common choice

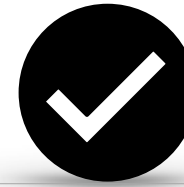


METHODOLOGY AND VALIDATION

- How can we include MHOU in a PDF fit?
- Can we validate our estimation?

*“Truth has nothing to do with the conclusion, and everything to do with the methodology”
(Stefan Molyneux)*

MHOU in a PDF fit: the *theory covmat*.



→ **Minimization** of a *loss function* depending on

Also, for a NNPDF fit...

→ Fit through a *Neural Network*

→ Delivery of a Montecarlo sample of fits on *replicas*



How to use it

FIT WITHOUT THEORY ERRORS

$$\chi^2 \propto (D_i - T_i) C_{ij}^{-1} (D_j - T_j)$$

$$\text{Pseudodata replica} \propto C$$

FIT WITH THEORY ERRORS

$$\chi^2 \propto (D_i - T_i) (C + S)_{ij}^{-1} (D_j - T_j)$$

$$\text{Pseudodata replica} \propto C + S$$

MHOU in a PDF fit: the *theory covmat*.

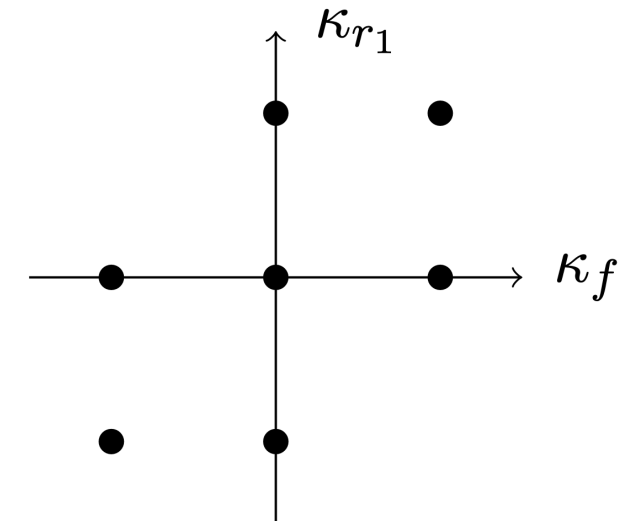
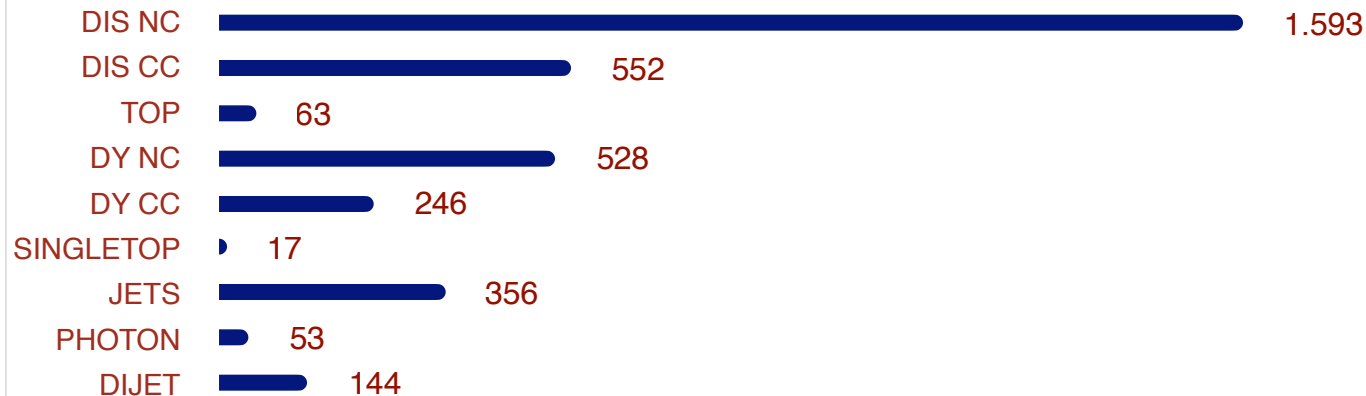


$$S_{ij} = n_m \sum_{V_m} \left(\bar{F}(\kappa_f, \kappa_{r_a}) - F \right)_{i_a} \left(\bar{F}(\kappa_f, \kappa_{r_b}) - F \right)_{j_b}$$

How to construct it

→ Factorization scale **correlates** all the points

→ Renormalization scale **correlates** points belonging to the same process



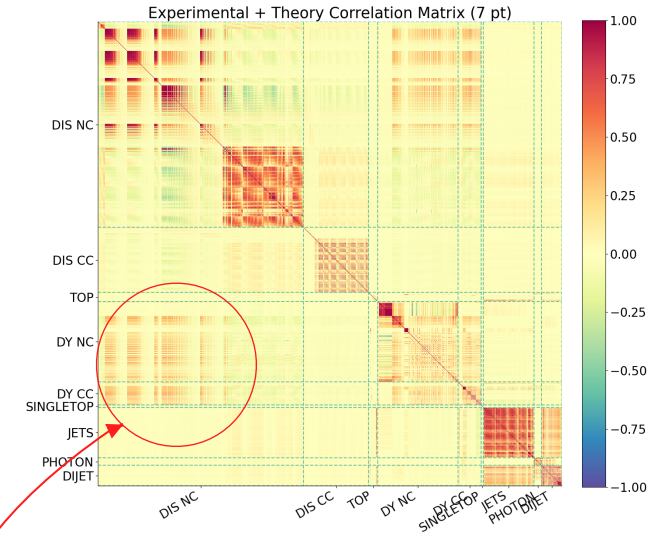
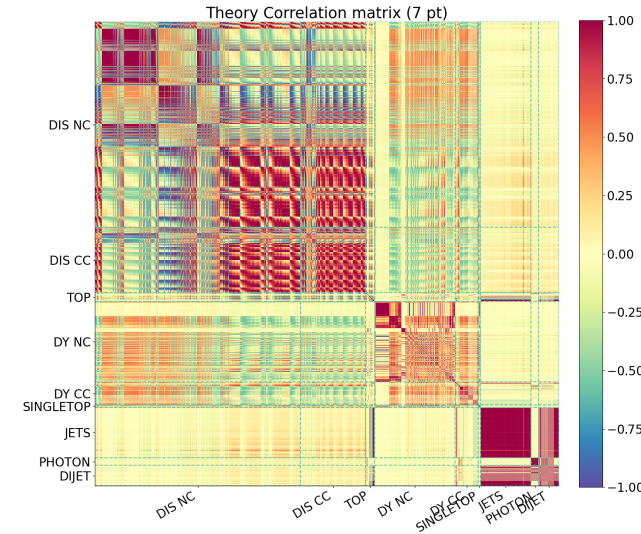
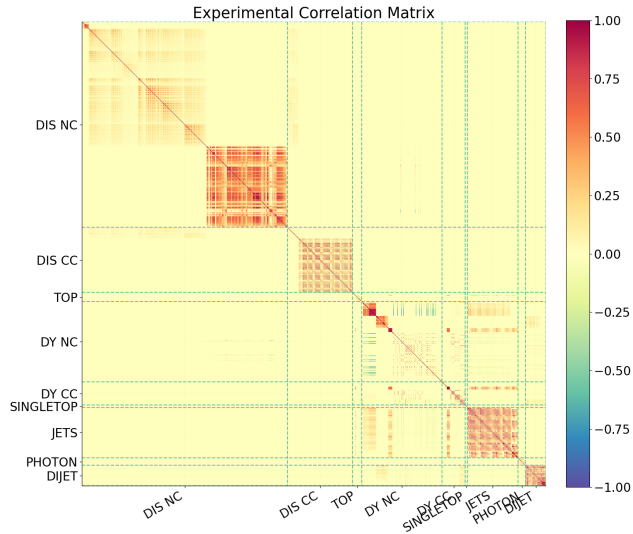
How do they look like?

↓
 C

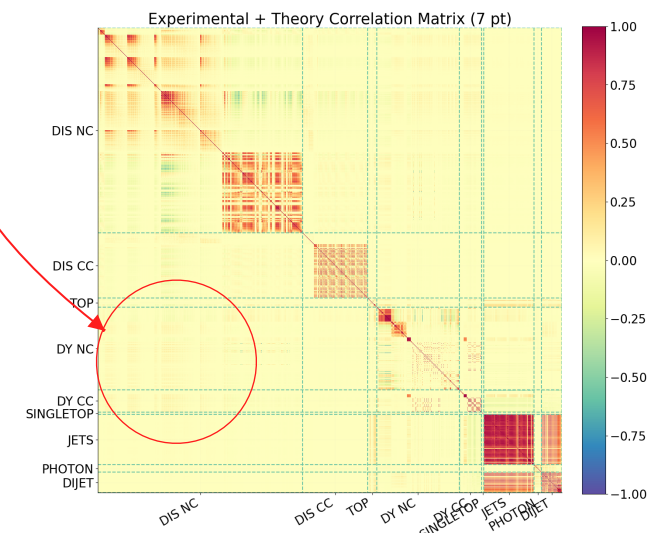
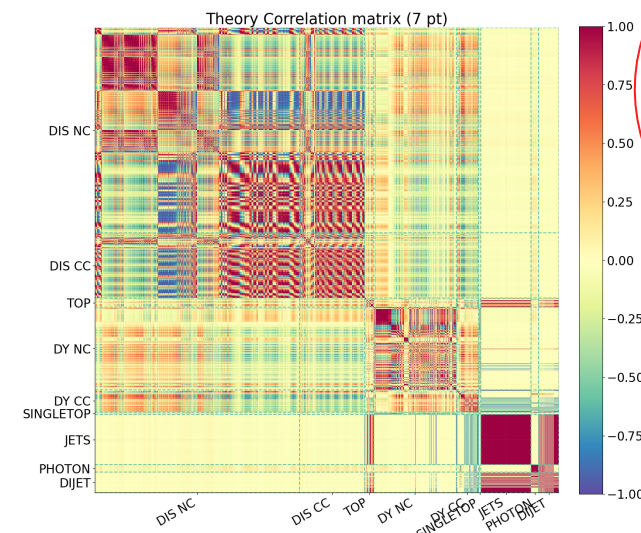
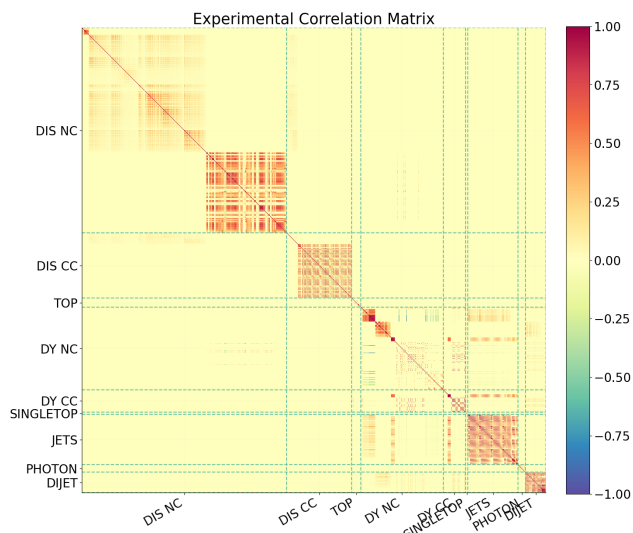
↓
 S

↓
 $C + S$

→ NLO

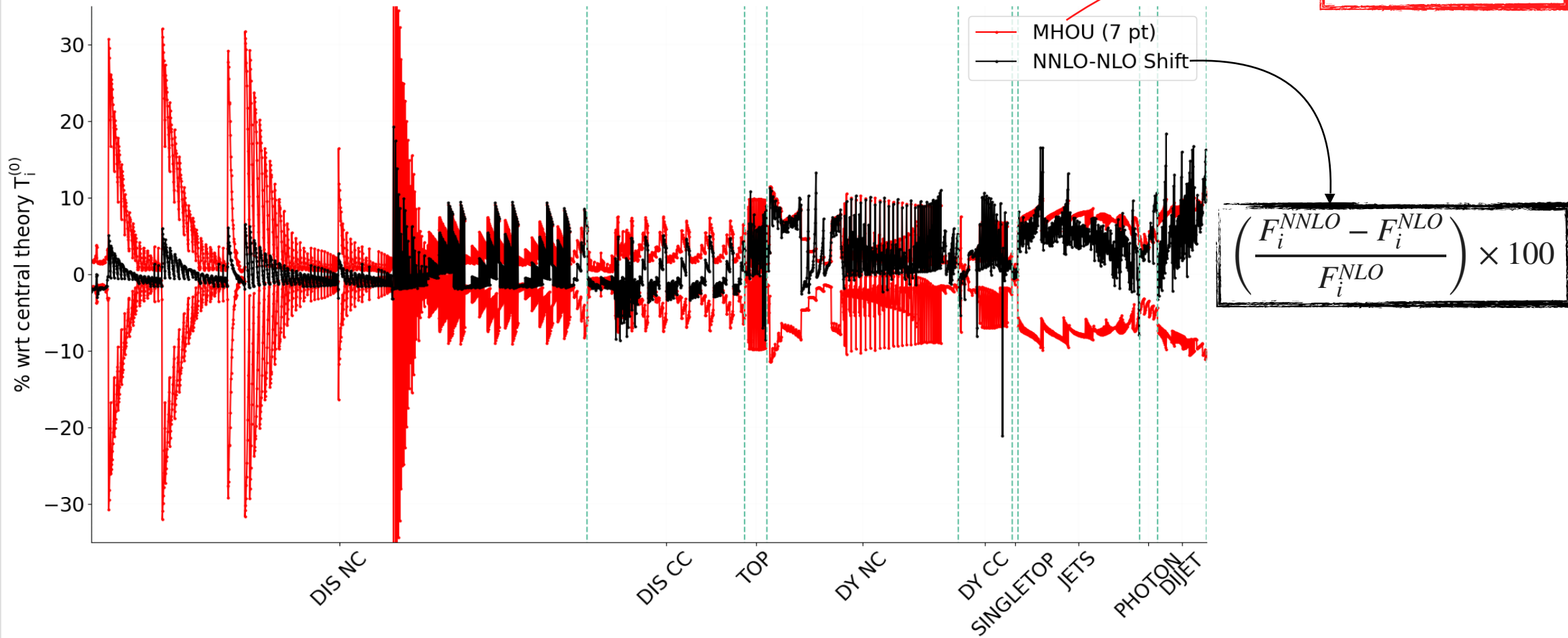


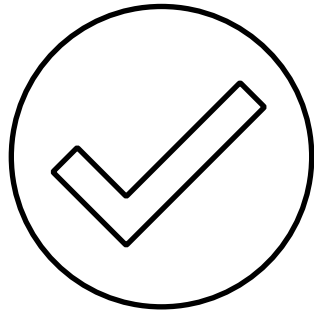
→ $NNLO$



Validation: is it reproducing the known MHOUs?

Most of the predictions are currently known up to $\mathcal{O}(NNLO)$:
we can test the NLO MHOUs !





RESULTS

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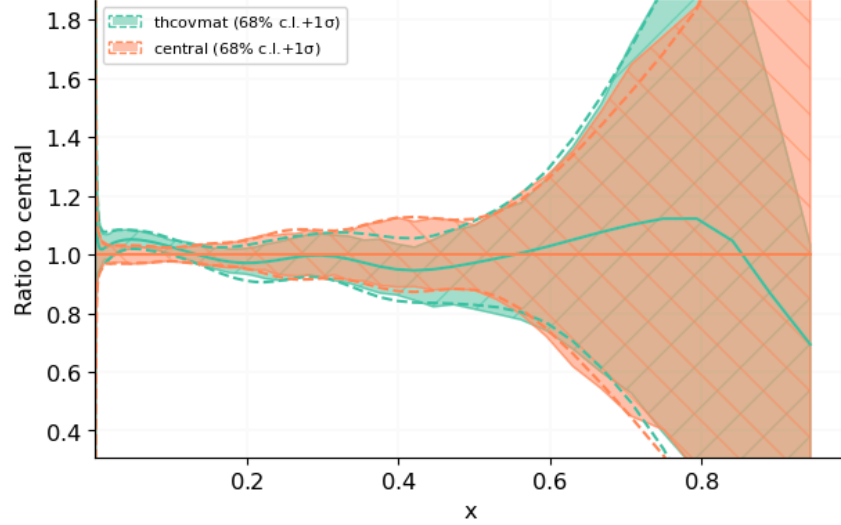
“We're always, by the way, in fundamental physics, always trying to investigate those things in which we don't understand the conclusions. After we've checked them enough, we're okay”

(Richard P. Feynman)

PDF central values change: NLO.

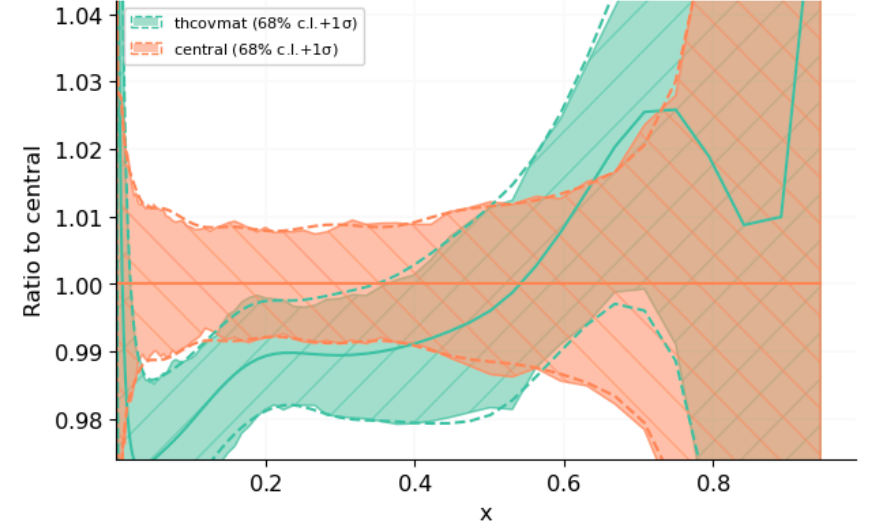
g

g at 1.651 GeV



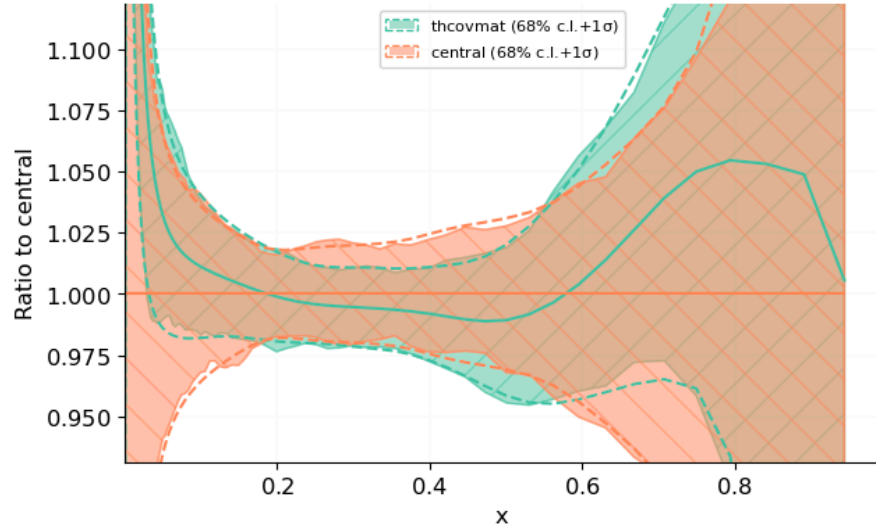
Σ

Σ at 1.651 GeV



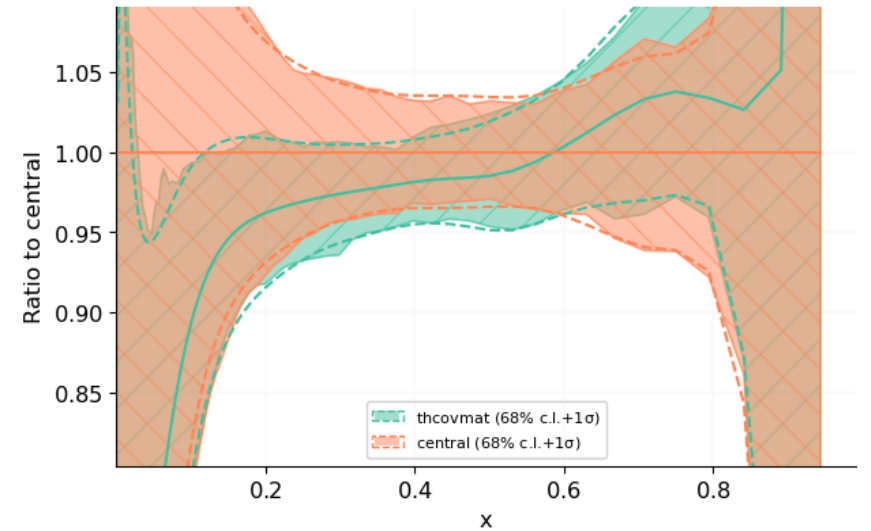
V

V at 1.651 GeV



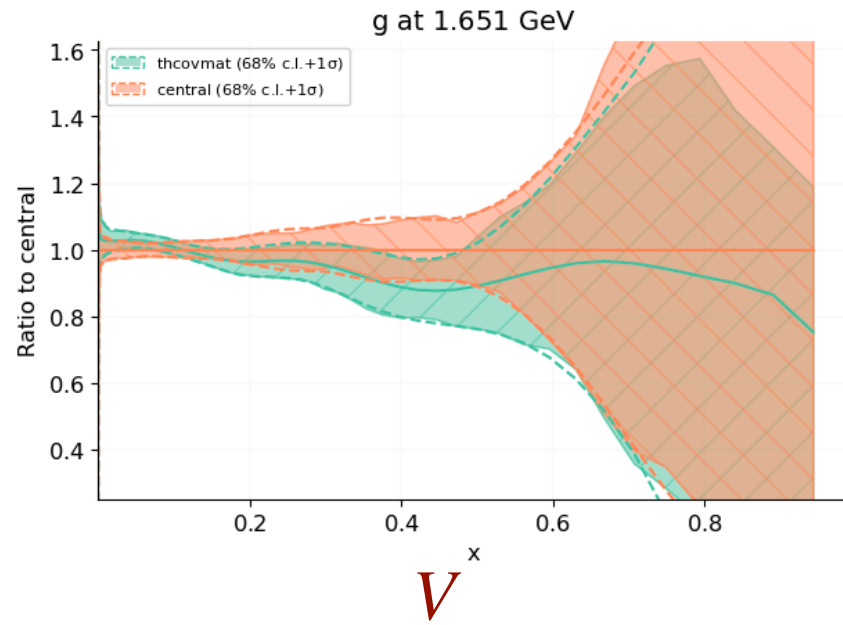
$T3$

$T3$ at 1.651 GeV

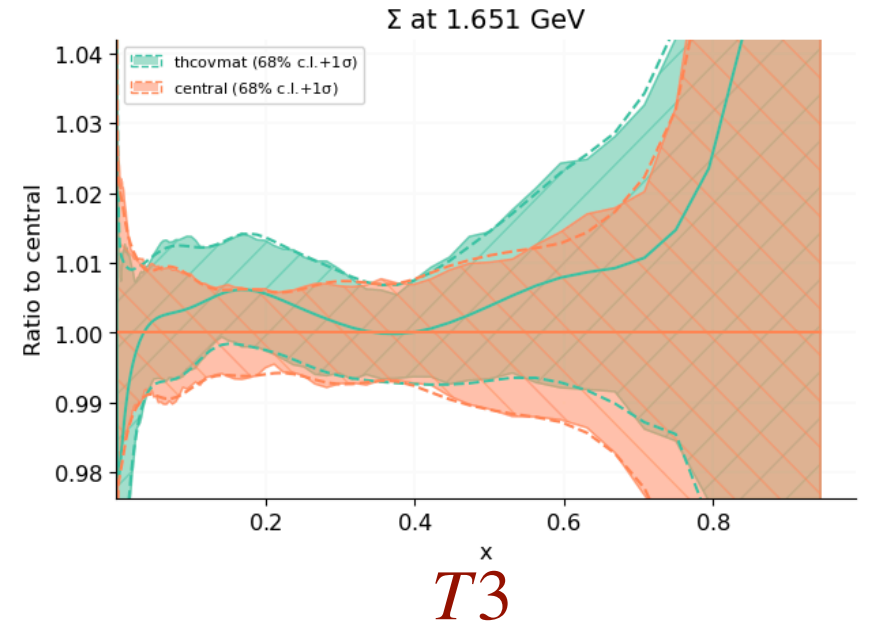


PDF central values change: NNLO.

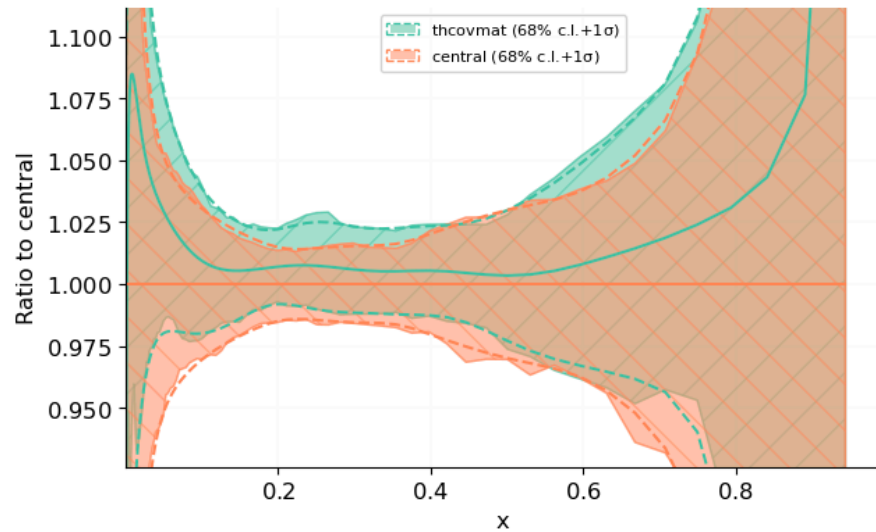
g



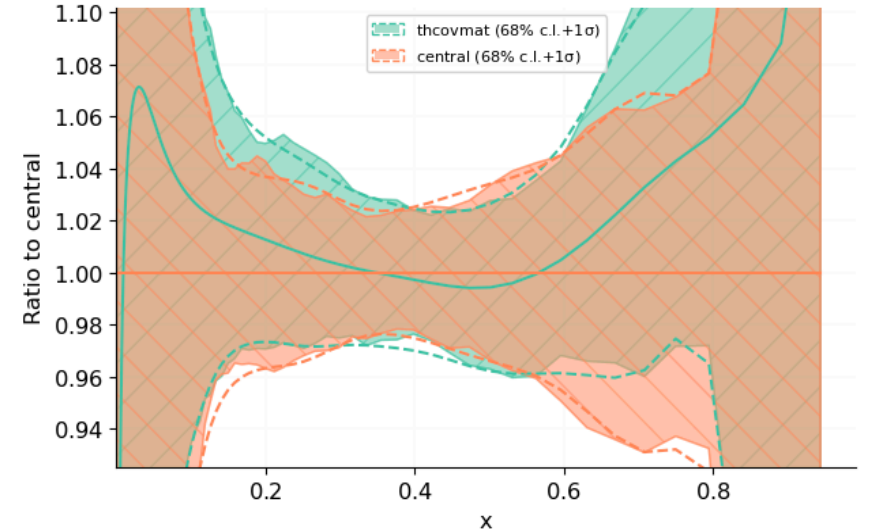
Σ



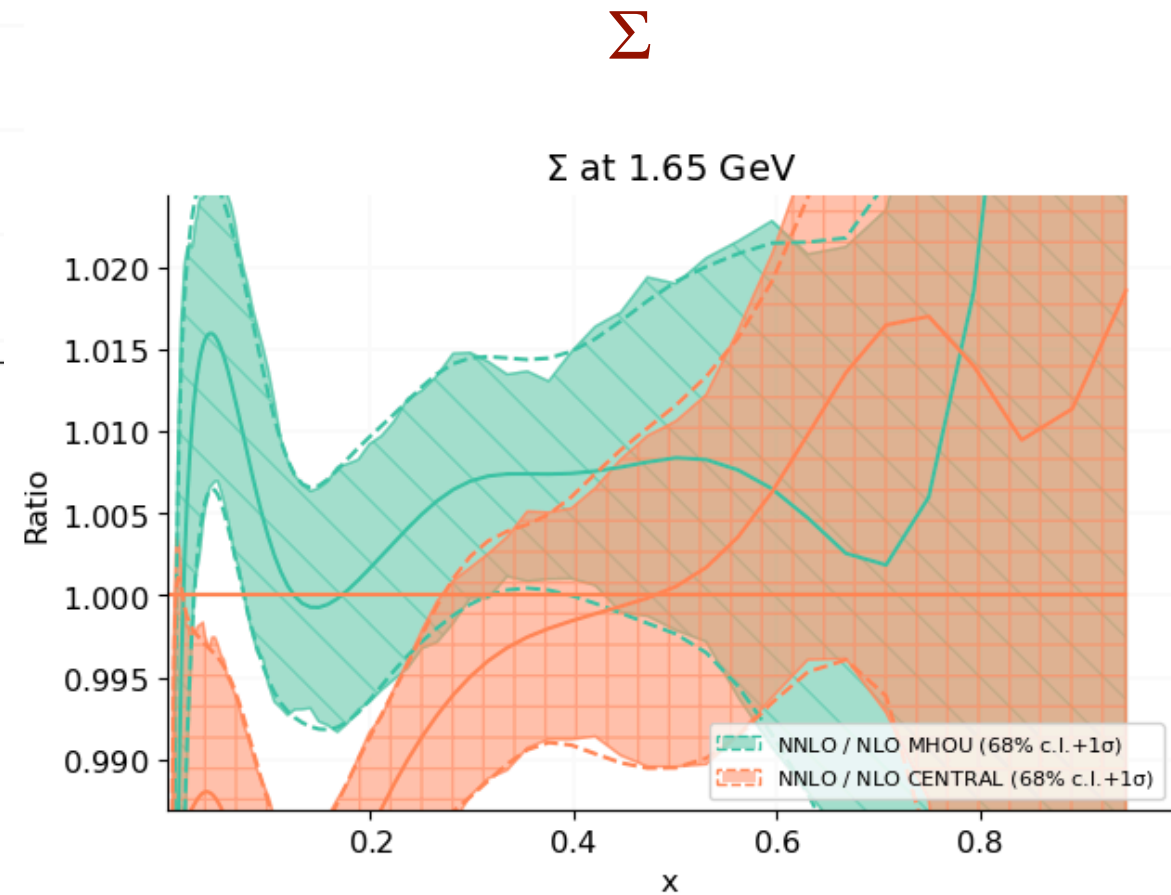
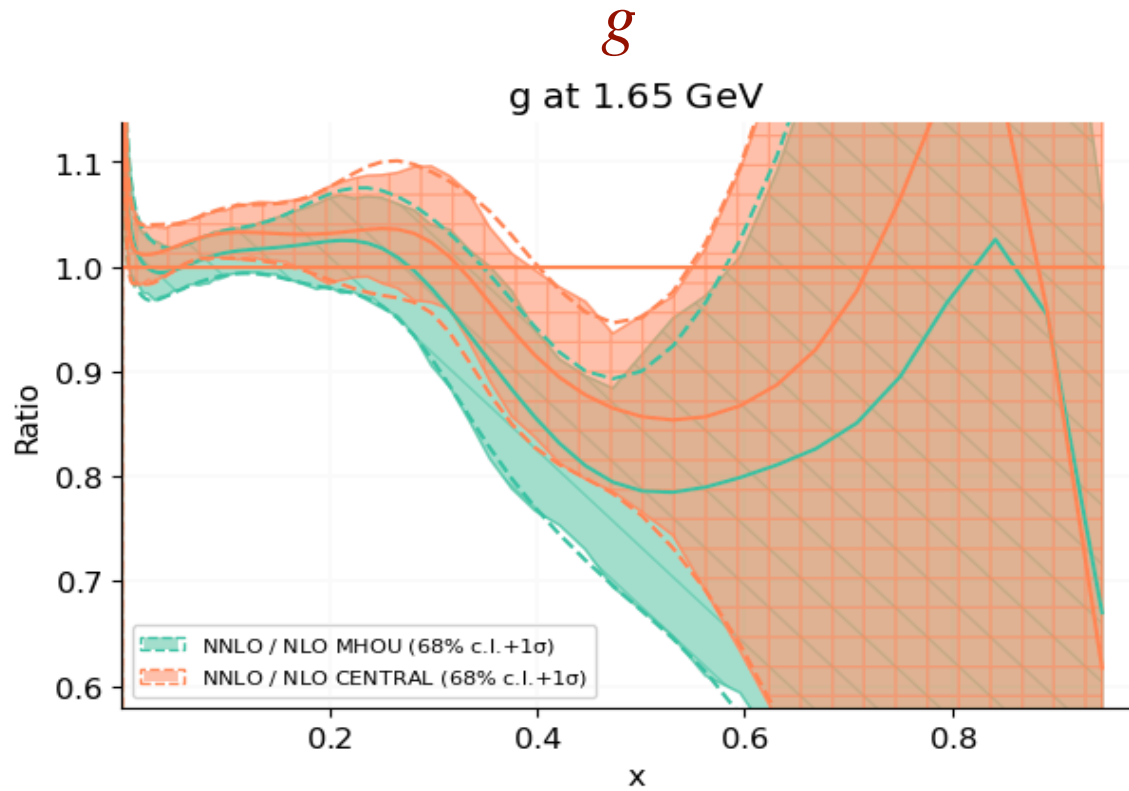
V at 1.651 GeV



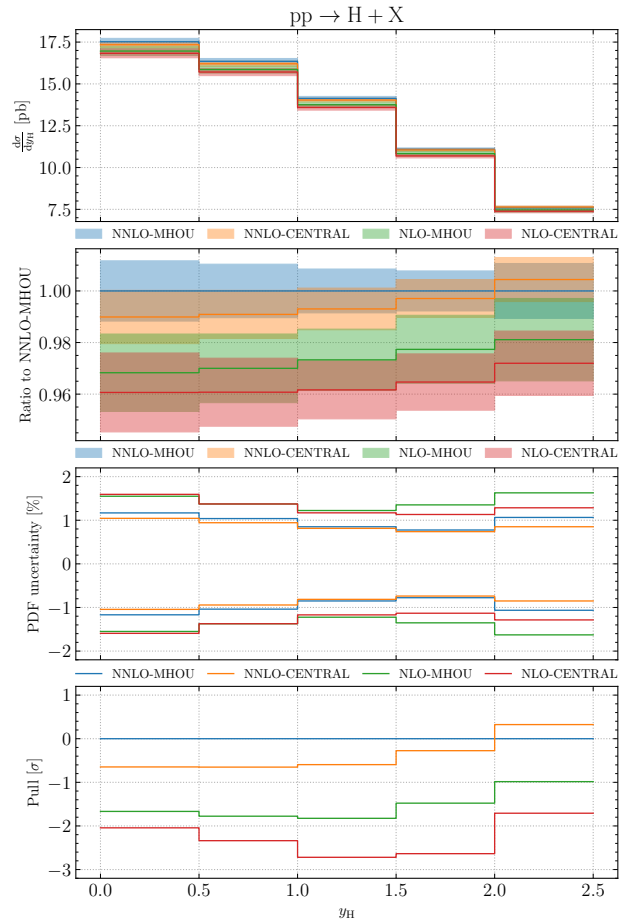
T3 at 1.651 GeV



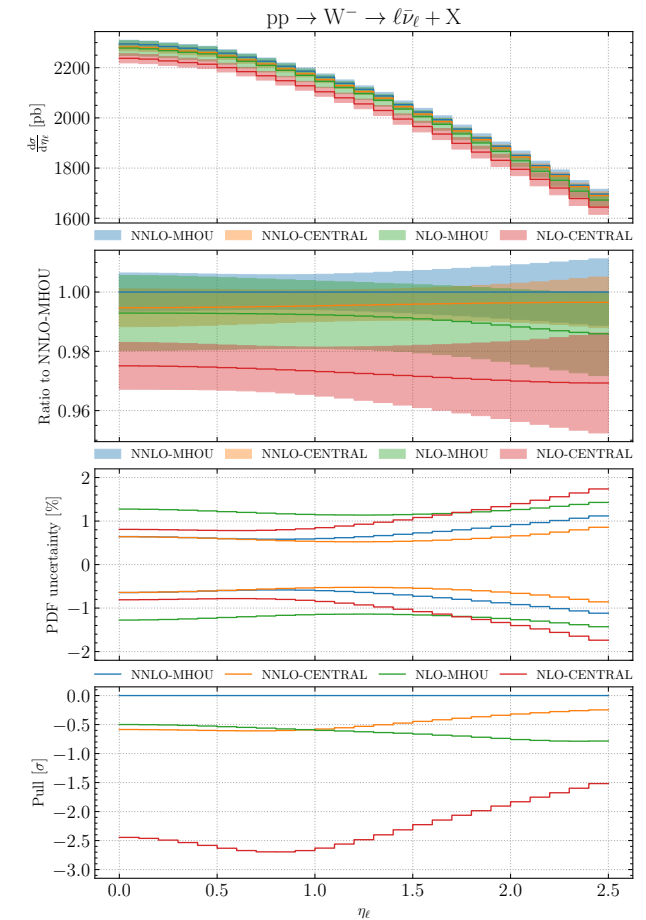
The perturbative convergence : NLO vs NNLO.



PDFs comparison in phenomenology.



NLO-MHOU is closer to NNLO than NLO-CENTRAL!



Using these MHOU PDF sets still requires the addition of the usual MHOU in the **partonic cross sections**

Conclusions.

- Thanks to *scale variations* it is possible to estimate MHOUs while, thanks to the theory covmat formalism, it is possible to include such estimation in a PDF fit
- Including MHOUs in a PDF fit is necessary to have faithful uncertainties and central values
- The perturbative convergence from NLO to NNLO improves once theory errors are accounted for

Thanks for your attention!